The Mean Age at Death (MAD): An alternative to life expectancy?

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Abstract

In this paper, we compare and discuss the standardized mean age at death (MAD) with the period life expectancy (LE) indicator. Since the observed MAD is affected by the population's age structure it is not an adequate metric for comparing mortality levels between populations. Both, the standardized MAD and period LE are free of the population's age structure and thus, appropriate for comparing mortality levels across population and over time. The standardized MAD, however, might be conceptually closer to the real population because it standardizes by controlling for fluctuations in births. Period LE, on the other hand, relies on the synthetic cohort approach. We first discuss and compare both measures formally. In the empirical part of the paper, we provide estimates for the observed MAD, standardized MAD, and period LE in Switzerland, Sweden, Denmark, and Portugal from 1990 to 2020. We find that the observed MAD and standardized MAD are currently very similar to each other, while period LE suggests higher values for the average length of life. Yet, period LE is the only measure reacting to the sudden increase in death rates observed in 2020. Thus, our preliminary results suggest that period LE is indispensable for examining period shocks in age-specific mortality rates. Still, the standardized MAD can be a valuable alternative to period LE for measuring the average length of life in the real population.

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1 Introduction

The most straight-forward way to inform policy makers and the general public about levels and trends in longevity is probably providing estimates of the mean age at death (MAD) observed in a given period (e.g., in a calendar year). From a statistical point of view, however, MAD is not an appropriate mortality measure because it is affected by the population's age structure; a population with a comparatively old age structure will show a higher MAD than a younger population even though mortality levels are identical in both populations. Demographers have addressed this issue by either relying on age-standardized measures such as the the age-standardized crude death rate (ASCDR) or by constructing the period life table model. The latter might be more attractive as it does not require choosing a standard population. The life table model assumes that a hypothetical cohort is subjected to the observed age-specific mortality rates over its whole life course. From this model, valuable statistics such as the MAD in the period life table population can be derived. This MAD is widely known as period life expectancy (LE) and a prominent tool for comparing mortality levels over time and across populations. However, the period life table populations differs usually substantially from the actual population, making LE difficult to interpret in the context of someone's average length of life. In this sense, the indicator might even cause more confusion than clarification (Luy et al. 2020). Another way to obtain a standardized mortality measure is assuming a population with a constant inflow of annual births which is closed to migration (Bongaarts and Feeney 2003). Accordingly, the age structure of this population is only shaped by cohort-specific mortality rates. The MAD observed in a given year can be interpreted as the population's mean age at death after controlling for changes in the initial size of cohorts (Guillot 2006). In accordance with the previous literature, we refer to this measure as the standardized MAD.

In this paper, we argue that the standardized MAD is a valuable alternative to LE because it is conceptually closer to the real population. We demonstrate this by discussing the three measures (observed MAD, period LE, and standardized MAD) formally and provide empirical time trends for Switzerland, Sweden, Denmark, and Portugal from 1990 to 2020.

2 Method

The observed MAD at time t can be calculated as:

$$MAD(t) = \frac{\int_0^\omega a \cdot D(a,t)da}{\int_0^\omega D(a,t)da}, \tag{1}$$

with $D(a,t)$ being the number of deaths at age a in time t.

The age-specific number of deaths in time t is given by the force of mortality $\mu(a,t)$ and the population size $N(a,t)$:

$$D(a,t) = N(a,t) \cdot \mu(a,t). \tag{2}$$

This equation reveals the effect of the population structure on $D(a,t)$. A constant $\mu(a,t)$ can lead to different death distributions, depending on the population's age structure $N(a,t)$.

As mentioned above, the effect of the population structure on MAD hinders comparability. For this reason, demographers either select a standard population for $N(a,t)$ or rely on the period life table. In the period life table, the age-specific number of life tables deaths $d(a,t)$ is given by:

$$d(a,t) = l(a,t) \cdot \mu(a,t), \tag{3}$$

where $l(a,t)$ denotes the period life table survival function. The MAD in the period life table (or period LE) can be calculated as:

$$LE(t) = \frac{\int_0^\omega a \cdot d(a,t)da}{\int_0^\omega d(a,t)da}, \tag{4}$$

Interestingly, $l(a,t)$ is a function of $\mu(a,t)$ itself,

$$l(a,t) = e^{-\int_0^\omega \mu(a,t)da}, \tag{5}$$

and can be seen as the age distribution of the stationary population implied by $\mu(a,t)$ (Preston, Heuveline, and Guillot 2001). In real populations, $N(a,t)$ and $l(a,t)$ differ considerbly from each other. This is because $N(a,t)$ is a function of past births and cohort survival,

$$N(a,t) = B(t-a) \cdot p_c(a,t-a), \tag{6}$$

where $B(t-a)$ is the number births a years before time t and $p_c(a,t-a)$ is the probability for individuals being born in time $(t-a)$ to survive up to age a. The $l(a,t)$ function, on the other hand, refers to the age structure of a population that experiences current mortality rates for about 100 years. In other words, $N(a,t)$ is clearly a function of the past, while $l(a,t)$ refers more to a scenario in the future.

The standardized MAD uses the standardized age distribution $S(a,t)$ for deriving the age-specific death counts, which has been called the standardized death distribution $D_{Std}(a,t)$:

$$Std.\ MAD(t) = \frac{\int_0^\omega a \cdot D_{Std}(a,t)da}{\int_0^\omega D_{Std}(a,t)da}, \tag{7}$$

and

$$D_{Std}(a,t) = S(a,t) \cdot \mu(a,t), \tag{8}$$

where $S(a,t)$ refers to the age distribution that standardizes $N(a,t)$ for fluctuations in births by holding the number of births constant over time,

$$S(a,t) = B \cdot p_c(a,t-a). \tag{9}$$

3 Preliminary results

The three age-specific death distributions are depicted in figure 1 for men in Switzerland in 2019 and 2020. The total death count has increased substantially between 2019 and 2020 but the deaths were

mostly observed at older ages, leading still to a small increase in the observed MAD between the two years (from 77.30 years in 2019 to 78.00 years in 2020). Looking at the standardized death distributions reveals the increase in MAD is slightly smaller after taking into account fluctuations in the number of births (from 78.34 years in 2019 to 78.86 years in 2020). LE, however, decreased between 2019 and 2020 from 81.89 years to 81.00 years. This comparisons reveals that the observed MAD and the standardized MAD indicate a relatively similar level for the average life span (both between 77-78 years), while LE suggest much more life years (about 81 years). This is because mortality has been decreasing over time and the age distribution implied by current mortality rates puts higher weights to older individuals than the observed and standarduzed age distribution, leading to a higher MAD. Something similar has been recently described by Modig, Rau, and Ahlbom (2020). Further, LE is more sensitive to changes in age-specific death rates and decreases between 2019 and 2020. It is, therefore, more appropriate for capturing period shocks in mortality (Rodríguez 2006).
The time trend from 1990 to 2020 for the three measures is shown in figure 2. For both, women and men LE is considerbly higher than the observed MAD and the standardized MAD. Further, MAD and standardized MAD do not necessarily show similar longevity levels. For instance, the MAD values for men in Sweden lay between the LE and standardized MAD estimates from 1990 to 2005. Only recently, both MAD measures show a similar level. The difference depends on how much the observed death distribution corresponds to the standardized death distribution.

4 References

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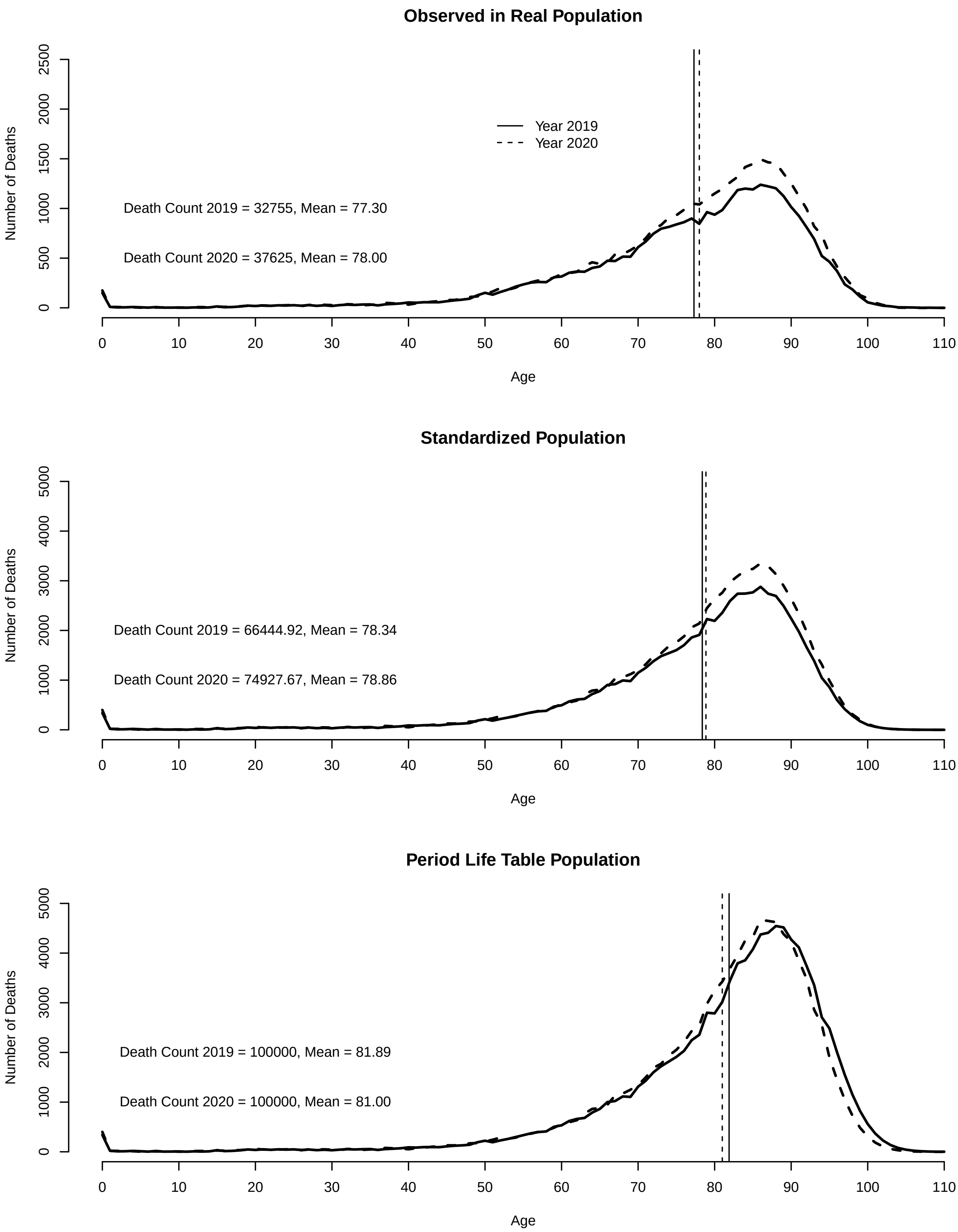
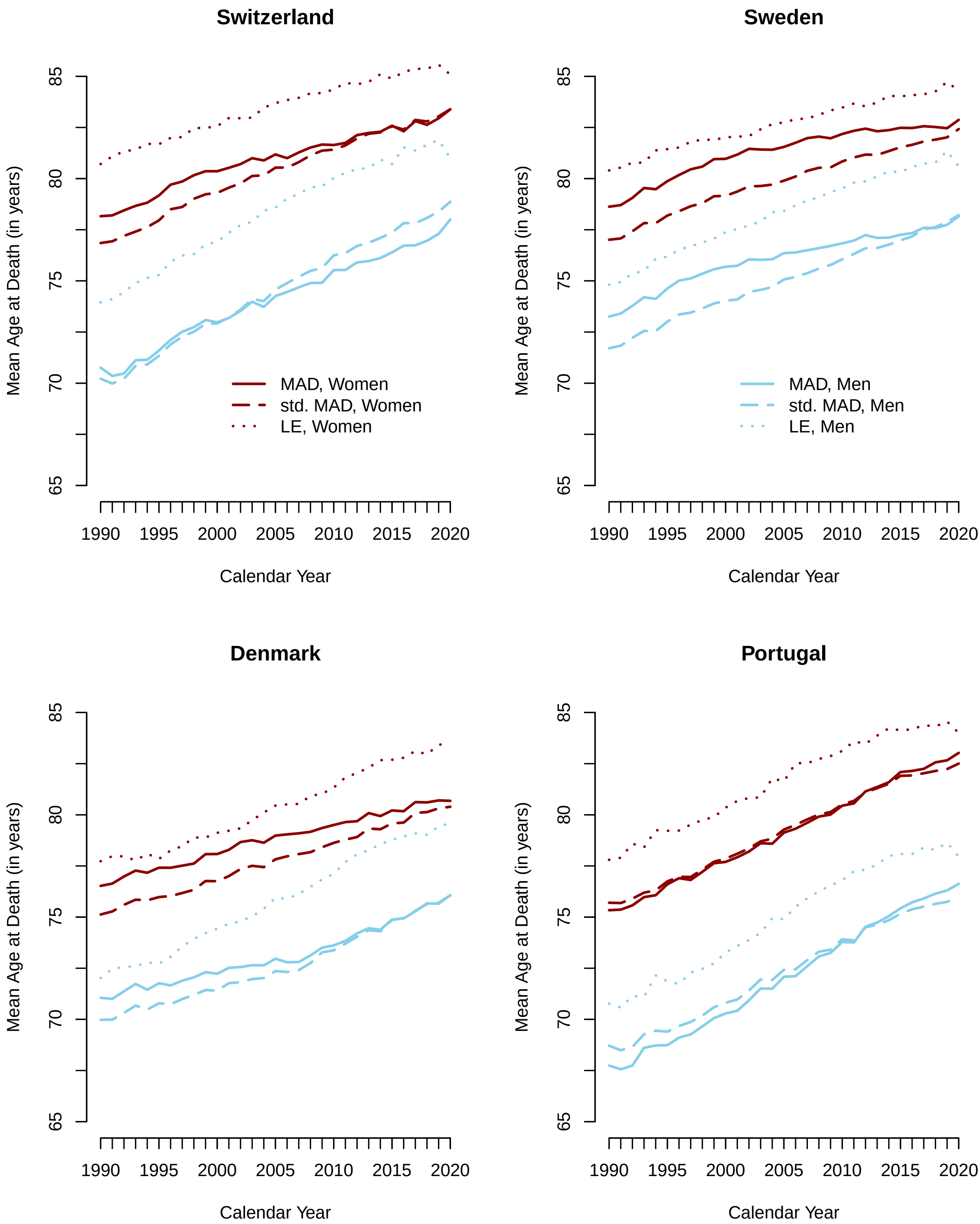
Figure 1: Observed death distribution, standardized death distribution, and period life table death distribution in Switzerland, 2019 and 2020, Men



Source: Human Mortality Database (2021).

Figure 2: Time series of the observed MAD, standardized MAD, and period LE for four selected countries from 1990 to 2020



Source: Human Mortality Database (2021).